O. P. JINDAL SCHOOL, SAVITRI NAGAR
Half Yearly Examination (2023-2024)

Class : XII **MM : 80**

Subject : Mathematics **Time Allowed : 3 Hrs.**

Name: _____ Class / Section: _____ Roll No.:_____

Fifteen Minutes Extra will be for reading the Question Paper.

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with 5 sub parts. You have to attempt any 4 parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION A

(This section comprises of very multiple choice questions (MCQ) of 1 mark each)

1. Let the functioin 'f' be defined by $f(x) = 5x^2 + 2 \; \forall \; x \in R$, then 'f' is

(a) onto function

(b) one-one, onto function

(c) one-one, into function

(d) many-one into function

OR

If $f : R \rightarrow R$ such that $f(x) = 3x$ then what type of a function is f?

(a) one-one onto

(b) many one onto

(c) one-one into

(d) many-one into

2. A relation R in human being defined as, $R = \{\{a, b) : a, b \in$ human beings : a is brother of b$\}$ is-

(a) reflexive

(b) symmetric and transitive

(c) equivalence

(d) None of these

3. The relation $R = \{(1,1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is

(a) Reflexive but not symmetric

(b) Reflexive but not transitive

(c) Symmetric and transitive

(d) Neither symmetric nor transitive

4. The domain of the function $f(x) = \sin^{-1}(x)$ is:

(a) $[-1, 1]$

(b) $(-\infty, \infty)$

(c) $[0, 1]$

(d) $[-\pi/2, \pi/2]$

5. The range of the function $f(x) = \tan^{-1}(x)$ is:
   (a) $(-\infty, \infty)$
   (b) $[0, \pi/2)$
   (c) $[-\pi/2, \pi/2]$
   (d) $(-\pi/2, \pi/2)$

6. The principal value of $\tan^{-1}[\tan(\frac{2\pi}{3})]$ is:
   (a) $\frac{2\pi}{3}$
   (b) $\frac{\pi}{3}$
   (c) $-\frac{\pi}{3}$
   (d) None of these

7. If A and B are two square matrices of order 3×3 such that $|A| = 2$ and $AB = 2I$, then the value of $|B|$ is :
   (a) 4
   (b) 8
   (c) 2
   (d) None of these

OR

If A and B are two matrices of order $3 \times m$ and $n \times 2$ respectively, then the order of matrix $(5A - 3B)$ is:
   (a) 3×2
   (b) 2×3
   (c) 3×3
   (d) 2×2

8. Total number of possible matrices of order 3×3 with each entry 2 or 0 is
   (a) 9
   (b) 27
   (c) 81
   (d) 512

OR

If a matrix is both symmetric matrix and skew symmetric matrix then
   (a) A is a diagonal matrix
   (b) A is zero matrix
   (c) A is scalar matrix
   (d) None of these

9. The value of x for which the matrix $\begin{bmatrix} 3+x & 5 \\ 4 & 6-x \end{bmatrix}$ is a singular matrix is:
   (a) 4
   (b) 2
   (c) 3
   (d) None of these

10. If $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$, then value of x is:
   (a) 1
   (b) -1
   (c) 0
   (d) 2

11. If $y = sin^{-1} x$, then $(1 - x^2)y_2$ *is* equal to
   (a) xy
   (b) xy_2
   (c) xy_1
   (c) x^2

12. The value of 'k' for which the function f(x) defined by

$$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$, is continuous at x = 0 is :

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{1}{4}$

(d) None of these

OR

The derivative $y = \log(\cos e^x)$ w.r.t. x is:

(a) $\tan e^x$

(b) $e^x \tan e^x$

(c) $-e^x \tan e^x$

(d) None of these

13. If $x = a(1 - \cos\theta)$, $y = a(\theta - \sin\theta)$, then value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ is :

(a) $\frac{1}{2}$

(b) - 1

(c) 1

(d) None of these

14. Which of the following functions is decreasing on $(0,\pi/2)$?

(a) sin 2x

(b) tan x

(c) cos x

(d) cos 3x

OR

If x is real, the minimum value of $x^2 - 8x + 17$ is

(a) -1

(b) 0

(c) 1

(d) 2

15. Find the maximum profit that a company can make, if the profit function is given by $P(x) = 41 + 24x - 18x^2$.

(a) 25

(b) 43

(c) 62

(d) 49

OR

It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Find the value of a.

(a) 100

(b) 120

(c) 140

(d) 160

16. Evaluate : $\int_{-\pi/2}^{\pi/2} \sin^5 x \, dx$

(a) 0

(b) -2

(c) 2

(d) None of these

17. Evaluate: $\int(\tan x + \sec^2 x)\, e^x\, dx$

(a) $e^x \tan x + C$
(b) $e^x \sec^2 x + C$
(c) $e^x \sec x + C$
(d) None of these

18. Evaluate: $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) π
(d) None of these

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A) : The function $y = [x(x-2)]^2$ is increasing in $(0, 1) \cup (2, \infty)$
Reason (R) : $\frac{dy}{dx} = 0$, when $x = 0, 1, 2$.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false.
(d) A is false but R is true.

20. Assertion (A) : If $A=\begin{bmatrix}0 & 1 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}$ then $|3A| = 9|A|$
Reason (R) : If A is a square matrix of order n then $|kA| = k^n |A|$

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each)

21. Show that the modulus function $f : R \to R$, given by $f(x) = |x|$ is neither one-one nor onto.

22. Find the principal value of $\tan^{-1}[\sin(-\frac{\pi}{2})]$.

OR

Check whether the function $f: N \to N$ defined as $f(x) = 2x$ is one-one and onto or not.

23. If $A=\begin{bmatrix}-1 & 4\\ 0 & 5\end{bmatrix}$, then find |Adj A|.

24. If $y = \sin^{-1}(\frac{1-x^2}{1+x^2})$, find $\frac{dy}{dx}$.

OR

Find $\frac{dy}{dx}$ if $y = \sin(\tan^{-1} e^x)$.

25. A stone is dropped into a quiet lake and waves move in circles at a speed of 5cm/sec. At the instant when the radius of the circular wave is 8cm, how fast is the enclosed area increasing?

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Show that the relation R defined by R = {(a,b): a – b is divisible by 3, a,b ϵ N } is an equivalence relation.

27. Find the value of $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

28. Express the following matrix as the sum of a symmetric and a skew symmetric matrix and verify your result:

$$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$$

29. If $x\sqrt{1+y}+ y\sqrt{1+x} = 0,\ for, -1 < x < 1,$
Prove that: $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

OR

If x = a(θ – sinθ), y = a(1 – cosθ), find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$.

30. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 5$ is:
Strictly increasing (b) Strictly decreasing

31. Evaluate: $\int_{-5}^{5}|x+2|\,dx$

OR

Evaluate: $\int \frac{2x-3}{(x^2-1)(2x+3)}dx$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. If $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, then find A^{-1} and hence solve the system of equations:

3x - 2y + 3z = 8, 2x + y - z = 1, 4x - 3y + 2z = 4

33. Prove that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.
Also find the maximum volume.

34. Evaluate: $\int_0^{\pi}\log(1+cosx)\,dx$

OR

Evaluate: $\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

35. Evaluate: $\int \frac{(3sinx-2)\,cosxdx}{13-cos^2x-7sinx}$ dx

SECTION E

(This section comprises of 3 case based questions of 4 marks each. Attempt any 4 in each case)

36. Case-Study 1: Read the following passage and answer the questions given below.

Area of a triangle whose vertices are (x_1,y_1),(x_2, y_2) and (x_3, y_3) is given by the determinant

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

Since, area is a positive quantity, so we always take the absolute value of the determinant Δ.
Also, the area of the triangle formed by three collinear points is zero.
Based on the above information, answer the following questions:

1. Find the area of the triangle whose vertices are (-2, 6), (3, -6) and (1, 5).
   (a) 30 sq. units
   (b) 35
   (c) 40
   (d) 15.5

2. If the points (2, -3), (k, -1) and (0, 4) are collinear, then find the value of k is :
   (a) 4
   (b) 7
   (c) 5
   (d) 6

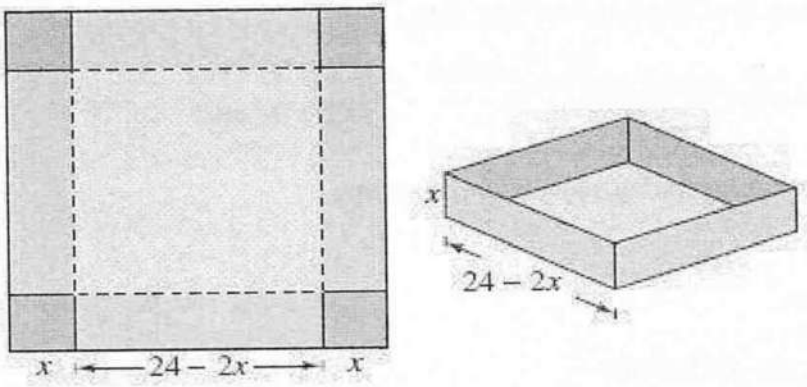
3. If the area of a triangle ABC, with vertices A(1, 3), B(O, 0) and C(k, 0) is 3 sq. units, then a value of k is
   (a) 2
   (b) 3
   (c) 4
   (d) 5

4. Using determinants, find the equation of the line joining the points A(1, 2) and B(3, 6).
   (a) $y = 2x$
   (b) $x = 3y$
   (c) $y = x$
   (d) $4x - y = 5$

5. If the area of a triangle ABC, with vertices A(5, 0), B(-1, 3) and C(1, 2) is 0, then point B lies
   (a) In the perpendicular bisector of AC
   (b) In the middle of AC
   (c) In the interior of triangle ABC
   (d) On the line joining A and C

37. Case-Study 2: Read the following passage and answer the questions given below.

A man has an expensive square shape piece of golden board of size 24 cm is to be made into a box without top by cutting from each corner and folding the flaps to form a box.



1. Volume of open box formed by folding up the flap of side x cm:
   (a) $4(x^3 - 24x^2 + 144x)$
   (b) $4(x^3 - 34x^2 + 244x)$
   (c) $x^3 - 24x^2 + 144x$
   (d) $4x^3 - 24x^2 + 144x$
2. In the first derivative test, if dy/dx changes its sign from positive to negative as x increases through c_1, then function attains a:
   (a) Local maxima at $x = c_1$
   (b) Local minima at $x = c_1$
   (c) Neither maxima nor minima at $x = c_1$
   (d) None of these
3. What should be the side of the square piece to be cut from each corner of the board to hold the maximum volume?
   (a) 14 cm
   (b) 12 cm
   (c) 4 cm
   (d) 5 cm
4. In the second derivative test, if d^2y/dx^2 is positive at critical point $x = c_1$, then function attains a:
   (e) Local maxima at $x = c_1$
   (f) Local minima at $x = c_1$
   (g) Neither maxima nor minima at $x = c_1$
   (h) None of these
5. What should be the maximum volume of open box?
   (a) 1034 cm^3
   (b) 1024 cm^3
   (c) 1204 cm^3
   (d) 4021 cm^3

38. Case-Study 3: Read the following passage and answer the questions given below.

Let f(x) be a continuous and a differentiable function.

If f(-x) = f(x), then f(x) is an even function.

And if f(-x) = -f(x), then f(x) is an odd function.

Again we have, $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)dx, & if\ f(x) is\ even \\ 0, & if\ f(x) is\ odd \end{cases}$

Based on the above information, answer the following:

1. $f(x) = x^2 \sin x$ is :
   (a) Even function
   (b) Odd function
   (c) Neither even nor odd function
   (d) None of these

2. $\int_{-a}^{a} f(x)dx$ is equal to
   (a) 2π
   (b) $\frac{\pi}{2}$
   (c) $\frac{\pi}{4}$
   (d) 0

3. If $f(x) = x \sin x$, then $\int_{-a}^{a} xsinx\ dx$ is
   (a) π
   (b) 2π
   (c) 3π
   (d) 4π

4. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} |sinx|\ dx$ is equal to
   (a) 1
   (b) 2
   (c) 3
   (d) 0

5. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} sin^7 x\ dx$ is equal to
   (a) 0
   (b) 1
   (c) 2
   (d) π